B.Sc. (Honours) Examination, 2021
Semester-III
Statistics
Course: CC 5
(Sampling Distribution)
Time: 3 Hours Full Marks: 40

Questions are of value as indicated in the margin
Notations have their usual meanings

Answer **any four** questions

1. a. State and prove Weak Law of Large Numbers (WLLN).
   b. Check whether Weak Law of Large Numbers (WLLN) holds for the following sequence of random variables:
   $i. P(X_n = n) = P(X_n = -n) = \frac{1}{2}n^{-\frac{1}{2}}, P(X_n = 0) = 1 - n^{-\frac{1}{2}}$
   $ii. P(X_n = -2^{n+1}) = P(X_n = 2^{n+1}) = 2^{-(n+3)}, P(X_n = 0) = 1 - 2^{-(n+2)}$
   4+6
2. a. Write down the test procedure to perform a large sample test for comparing two independent binomial proportions.
   b. Hence or otherwise find a $100(1-\alpha)\%$ confidence interval for the difference of proportions. Find the expected length of the interval. 6+4
3. a. Let $X_1$and X_2be independently binomially distributed random variables, with parameters $(n_1, \frac{1}{2})$ and $\left(n_2, \frac{1}{2}\right)$, respectively. Show that $X_1 - X_2 + n_2$ has the binomial distribution with parameters $(n_1 + n_2, \frac{1}{2})$.
   b. Let X and Y be independently distributed, each in the form $N(0,1)$. Show that $Z = X/Y$ has the Cauchy distribution with pdf
   $$f(z) = \frac{1}{\pi[1+z^2]}$$
   What would be the distributions of $W_1 = X/|Y|$ and $W_2 = X/|X|$? 4+6
4. (a) Let $X_1, X_2, \ldots\ldots, X_n$ follow $N(\mu, \sigma^2)$. Find the sampling distributions of sample mean $\bar{X}$ and sample variance S^2. Also show that $\bar{X}$ and S^2 are independently distributed.

   (b) If X and Y are independent are independent $N(0,1)$variables, show that $\frac{XY}{\sqrt{X^2+Y^2}}$ is distributed as $N(0, \frac{1}{4})$. 7+3
5. (a) Define χ^2distribution. Find its mean and variance. Prove the additive property of this distribution.
   (b) State and prove Lindeberg-Levy Central Limit Theorem (CLT). Hence or otherwise prove DeMoivre-Laplace theorem. 5+5

6. (a) Derive the pdf of an t-distribution.

(b) If X and Y are independent random variables each distributed uniformly over $(0,1)$, find the distributions of

(i) $\frac{X}{Y}$ (ii) XY (iii) $\sqrt{X^2 + Y^2}$ 4+6